

Influence of strong microwave radiation on static properties of a small Josephson junction

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Preliminary results are presented concerning static properties of a small Josephson junction under the influence of strong microwave radiation. We discuss the correspondence between a Brownian particle moving in a periodic potential and superconducting phase difference in a small Josephson junction. Next, we describe an experimental method of determining the amplitude of microwave current flowing across the junction. Typical examples of static characteristics of the junction are presented, including its dynamical resistance as a function of microwave power. We discuss also the influence of an external magnetic field on the junction dynamics and show that in this case the one-dimensional Stewart-McCumber model becomes insufficient.

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I. INTRODUCTION

Recently, the noise-assisted transport of Brownian particles has attracted considerable attention, which is still growing in view of possible applications in various branches of physics and chemistry.¹⁻³ Unfortunately, experimental investigation of transport phenomena on a molecular scale is extremely difficult, thus apart from theoretical studies there is a need for adequate physical systems, modeling various transport phenomena in a way which could be accessible experimentally.

As shown in Refs. 4,5, there is a rigorous mathematical equivalence between the Newton-Langevin (NL) equation, describing a Brownian particle in a periodic potential, and the Stewart-McCumber (SM) model dealing with the superconducting phase dynamics in a small Josephson junction in the presence of thermal fluctuations. In particular, a constant external force acting on a Brownian particle corresponds to a constant current biasing the Josephson junction, while a time-averaged velocity of the particle is equivalent to a constant voltage induced on the junction electrodes. In other words, the relevant parameters of a Brownian motor can be related to easily measurable macroscopic quantities such as a constant current and constant voltage, leading directly to the current-voltage (*I*-*V*) characteristic of the Josephson junction.

The aim of the present paper is twofold. First, we propose a uniform method making possible to determine the absolute value of microwave current amplitude between junction electrodes, independently of the junction geometry and its coupling to the experimental setup. Such a uniform approach covers a wide range of junction parameters, including two limiting cases of a “current source” and a “voltage source” model. Second, we investigate the influence of a strong microwave signal on the static properties of a small Josephson junction, with particular attention paid to its dynamic (differential) resistance. In this context we present preliminary results and explore the possibility of detecting a negative resistance (both absolute and differential) on the *I*-*V* characteristic.

The paper is organized as follows. In Sec. II we discuss the equivalence between the NL and SM models. Next, we recall the well-known relation between the critical current I_0 and the normal resistance R_n of a real Josephson junction, and discuss the possibility of making the junction parameters compatible with theoretical estimates. Experimental details, including coupling of the junction to a microwave setup, are given in Sec. III. Sec. IV deals with an experimental evaluation of rf (microwave) current flowing across the junction. In particular, we propose an analytical approximation making possible to determine the rf current amplitude for a wide range of junction parameters. A few examples of the current-voltage (*I*-*V*) characteristics are presented in Sec. V with a special attention paid to the dynamic resistance as a function of strong microwave signal. We discuss also the influence of an external magnetic field on the junction behavior and show that the superconducting phase difference ϕ becomes spatially modulated, making the SM model insufficient for a proper description of the junction dynamics. Sec. VI contains concluding remarks, we indicate also possible generalization to a multidimensional model of a Josephson junction immersed in an external magnetic field.

II. MODELING OF CHAOTIC PHENOMENA IN A JOSEPHSON JUNCTION

The chaotic dynamics of a classical Brownian particle in the presence of thermal noise can be described by the inertial NL equation:⁴

$$m\ddot{x} + \gamma\dot{x} + V'(x) = F + A \cos \Omega t + \sqrt{2\gamma k_B T} \xi(t) \quad (1)$$

where a dot and prime denote differentiation with respect to t and x , respectively. The particle mass is denoted by m , γ is the friction coefficient, Ω — the angular frequency, F and A denote constant and alternate external forces, respectively. Thermal fluctuations are described by zero-mean Gaussian white noise $\xi(t)$ with the autocorrelation function $\langle \xi(t)\xi(u) \rangle = \delta(t-u)$, k_B denotes the Boltzmann constant and T — temperature.

If a spatially periodic potential $V(x)$ is cosinusoidal, then it can be easily verified that Eq. (1) is formally equivalent to the SM model describing the superconducting phase dynamics of a one-dimensional small-area Josephson junction^{5,6}

$$\begin{aligned} \left(\frac{\hbar}{2e}\right) C \ddot{\phi} + \left(\frac{\hbar}{2e}\right) \frac{1}{R_n} \dot{\phi} + I_0 \sin(\phi) \\ = I_d + I_a \cos(\Omega t) + \sqrt{\frac{2k_B T}{R_n}} \xi(t) \end{aligned} \quad (2)$$

where ϕ denotes the superconducting phase difference between electrodes and the junction is characterized by its critical current I_0 , capacitance C and normal-state resistance R_n . The amplitudes of the dc and ac (microwave) currents applied to the junction are denoted by I_d and I_a , respectively.

A dimensionless form of Eq. (2) can be written as

$$\frac{d^2\phi}{dt'^2} + \sigma \frac{d\phi}{dt'} + \sin(\phi) = i_0 + i_1 \cos(\Omega_1 t') + \sqrt{2\sigma D} \Gamma(t') \quad (3)$$

where the time has been rescaled to $t' = t/\tau_0$, $\tau_0 = 1/\omega_p = \sqrt{\hbar C/2eI_0}$. Consequently, $\Omega_1 = \Omega\tau_0$, $\sigma = \tau_0/R_n C$, and the noise intensity $D = 2ek_B T/\hbar I_0$. The dimensionless dc and ac amplitudes are given by $i_0 = I_d/I_0$ and $i_1 = I_a/I_0$, respectively.

Comparison of Eq. (3) with Eq. (1) shows clearly that (after appropriate scaling of Eq. (1)) both models are mathematically equivalent, although the physical meaning of relevant variables is distinctly different. In particular the superconducting phase difference ϕ in Eq. (2) corresponds to the spatial coordinate x in Eq. (1). Consequently, the average value of the time-derivative $\langle \dot{\phi}_t \rangle$, proportional to the constant voltage across the junction, is analogous to the averaged velocity of a Brownian particle. Similarly, external current amplitudes I_d and I_a correspond to external forces F and A , respectively.

The range of dimensionless parameters in which we can expect either an absolute or a differential negative resistance, is given in Ref. 5. Thus, using the scaling of Eq. (3) and coming back to Eq. (2) it is possible to recover absolute values of the junction parameters and the external driving.

For example, the noise intensity $D = 10^{-3}$ implies $I_0 \simeq 176 \mu\text{A}$ at $T = 4.2 \text{ K}$. Moreover, assuming $\Omega = 150 \times 10^9$, i.e. $f \simeq 23.87 \text{ GHz}$ we obtain the following sets of junction parameters

$$\begin{aligned} (a) \quad R_n = 2.6 \Omega, \quad C = 13.8 \text{ pF}, \quad I_0 = 176 \mu\text{A} \\ (4) \end{aligned}$$

$$(b) \quad R_n = 1.4 \Omega, \quad C = 27.9 \text{ pF}, \quad I_0 = 176 \mu\text{A}$$

The external rf current amplitude may vary, ranging from $I_a \simeq 120 \mu\text{A}$ for the set (a) up to $I_a \simeq 500 \mu\text{A}$ for the set (b).⁵

The main difficulty with practical application of the above parameters follows from the fact that for a real

Josephson junction the normal resistance R_n and the critical current I_0 are not independent, but their product is given by:⁷

$$I_0 R_n = \frac{\pi \Delta(T)}{2e} \tanh \left(\frac{\Delta(T)}{2k_B T} \right) \quad (5)$$

where $\Delta(T)$ denotes the superconducting gap energy.

For niobium at $T = 4.2 \text{ K}$, we have $\Delta(T) \simeq 1.4 \text{ meV}$, thus the upper limit for $I_0 R_n$ can be estimated according to Eq. (5) as $I_0 R_n \simeq 2 \text{ mV}$. In practice, the value of $I_0 R_n$ for a real junction is lower due to inevitable defects, nevertheless it is still a few times greater than the value of $I_0 R_n$ resulting from the set of parameters (4) required by the theory.

In order to make real junction parameters compatible with theoretical predictions one can use one of two experimentally accessible methods, reducing either R_n or I_0 . In the first case, the effective value of normal resistance can be decreased by shunting the junction with an additional resistor connected in parallel to the junction electrodes.⁸ In this work, however, we use an alternative approach to reduce the critical current I_0 by applying an external constant magnetic field in the junction plane.

The following sets of junction parameters have been chosen for further studies:

$$\begin{aligned} (a) \quad R_n = 2.96 \Omega, \quad C = 14.0 \text{ pF}, \quad I_0 = 460 \mu\text{A} \\ (6) \\ (b) \quad R_n = 1.45 \Omega, \quad C = 28.0 \text{ pF}, \quad I_0 = 920 \mu\text{A} \end{aligned}$$

Comparing the above parameters with theoretical predictions (4) one can see that the values of R_n and C correspond approximately to those specified previously, while the nominal critical currents I_0 are a few times greater than $I_0 \simeq 176 \mu\text{A}$ required by the theory. As mentioned above, the critical current can be adjusted by applying an external magnetic field to the junction. In this manner, we are able to obtain the relevant junction parameters in agreement with those predicted theoretically.

The amplitude I_a of the rf current flowing across the junction is obviously proportional to \sqrt{P} , where P denotes the incident microwave power. Unfortunately, I_a depends in a rather complicated way on the geometry of the sample holder and its coupling to the microwave source. Therefore, the absolute value of I_a has been determined experimentally by using the well-known relations between the effective constant current at zero voltage and the rf current amplitude I_a (see Section IV for details).

III. EXPERIMENTAL

The Nb-AlO_x-Nb Josephson junctions in overlap geometry have been designed and manufactured in the Institute of Radio Engineering and Electronics in Moscow.

The sample holder was designed as a section of a ridge waveguide of reduced impedance, which was used to improve matching of the junction to the standard K -band (WR 42) waveguide and obtain a relatively large amplitude I_a of the rf current. Additionally, I_a could be controlled by a variable short located just behind the sample holder. Two superconducting coils of nearly rectangular cross-section were attached to the side walls of the sample holder, and could be used to apply a dc magnetic field of up to ~ 40 Gs directed parallel to the junction plane.

The measurement system consisted of a K -band klystron, followed by a ferrite isolator, ferrite modulator and calibrated attenuator. The microwave frequency could be controlled in the range 23 – 25 GHz and the maximum output power level was estimated as 100 mW.

The current-voltage (I - V) characteristics were measured by a standard 4-point technique. The junction was biased by a digital current source while the voltage was measured using a digital voltmeter and recorded by a computer. All the measurements were carried out at the temperature $T = 4.2$ K.

IV. EVALUATION OF THE MICROWAVE CURRENT AMPLITUDE

According to the “current source” model⁹ we consider the following equation:

$$\frac{d\phi}{d\tau} + \sin(\phi) = i_0 + i_1 \sin(\xi\tau), \quad (7)$$

where $i_0 = I_d/I_0$, $i_1 = I_a/I_0$ as before, $\xi = \Omega/\omega_c$ and $\omega_c = 2eR_nI_0/\hbar$. As compared to Eq. (3), we use a slightly different time scaling $\tau = \Omega_c t$ and neglect both the second time-derivative and the noise term.

Eq. (7) has been solved numerically in several papers^{7,9–11} and it has been shown that the maximum constant current I_{max} at zero voltage decays with increasing rf amplitude i_1 . However, detailed behavior of I_{max} as a function of i_1 depends strongly on the frequency parameter ξ . For $\xi \ll 1$, we have a “current source” model, I_{max} decreases linearly and $I_{max} = 0$ is attained for $i_1 \simeq 1$. On the other hand, for $\xi \gg 1$ the model tends to the “voltage source” with a Bessel-like behavior:

$$I_{max}/I_0 = J_0\left(\frac{2eR_nI_a}{\hbar\Omega}\right), \quad (8)$$

where J_0 denotes the Bessel function of order 0 and the argument can be transformed to a simpler form by using dimensionless quantities:

$$\frac{2eR_nI_a}{\hbar\Omega} = \frac{\omega_c i_1}{\Omega} = \frac{i_1}{\xi}. \quad (9)$$

Hence, asymptotically $I_{max} = 0$ is obtained at $i_1 = \kappa_{0,1}\xi$, where $\kappa_{0,1} = 2.405$ denotes the first zero of the Bessel function J_0 . Fig. 1 shows the results of numerical

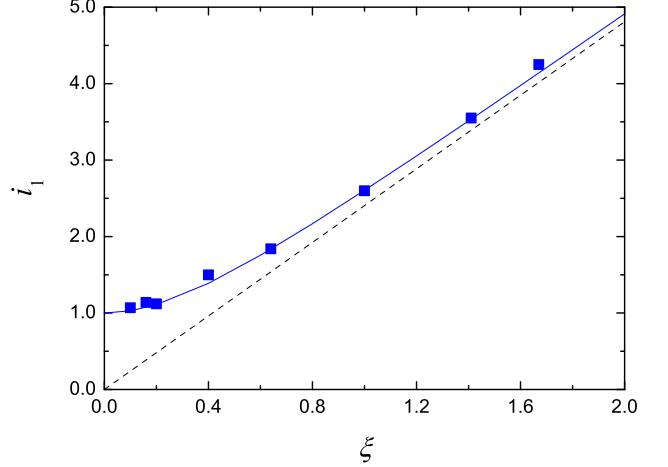


FIG. 1: (Color online) The relative rf current amplitude i_1 at $I_{max} = 0$ plotted as a function of the normalized frequency ξ . Full squares denote numerical results, the dashed line shows an asymptotic linear dependence, and the solid line represents an analytical approximation (10).

calculations of $i_1|_{I_{max}=0}$ as a function of ξ , taken from Refs. 7,9–11. The dashed line denotes an asymptotic linear dependence $i_1 = \kappa_{0,1}\xi$ and the solid line shows an analytical approximation suggested here to cover the whole range of ξ :

$$i_1 = \sqrt{1 + (\kappa_{0,1}\xi)^2}. \quad (10)$$

It is clear that the above approximation exhibits a proper behavior both for $\xi \rightarrow 0$ and $\xi \gg 1$. Moreover, as shown in Fig. 1, the analytical approximation (10) is in good agreement with numerical data, thus it can be used to evaluate the rf current amplitude i_1 also for intermediate values of ξ .

As an example, Fig. 2 shows an experimental dependence of I_{max} on the relative amplitude of microwave signal $(P/P_0)^{1/2}$, where P/P_0 denotes the relative microwave power.

The junction parameters are slightly different from those given in Eq. (6) and are equal to $R_n = 2 \Omega$, $I_0 = 680 \mu\text{A}$, hence $\xi \simeq 0.037$ for the microwave frequency $f \simeq 24$ GHz, and according to the analytical approximation (10) we find $i_1|_{I_{max}=0} \simeq 1.004$. It follows from Fig. 2 that $I_{max} = 0$ for $(P/P_0)^{1/2} \simeq 0.45$ what means that the absolute amplitude of the rf current at this point is equal to $I_a \simeq I_0 = 680 \mu\text{A}$. Thus, the maximum available current amplitude can be evaluated as $I_a^{(max)} = I_a/0.45 \simeq 1.5 \text{ mA}$ and is far above the range of I_a predicted by theoretical considerations (see Section II). Strictly speaking, the above analysis deals with the RSJ model, since the second time-derivative has been neglected in Eq. (7). The influence of the junction capacitance on the rf current amplitude is not fully understood, however as follows from Ref. 11, the estimates taking into account a nonzero capacitance yield even larger values of I_a .

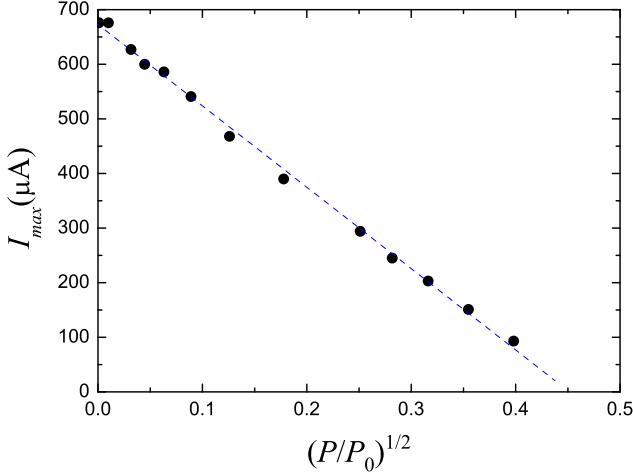


FIG. 2: (Color online) Experimental dependence of the maximum constant current I_{max} (full circles) on the relative amplitude of the microwave signal $(P/P_0)^{1/2}$. The dashed line shows a linear fit to the experimental data.

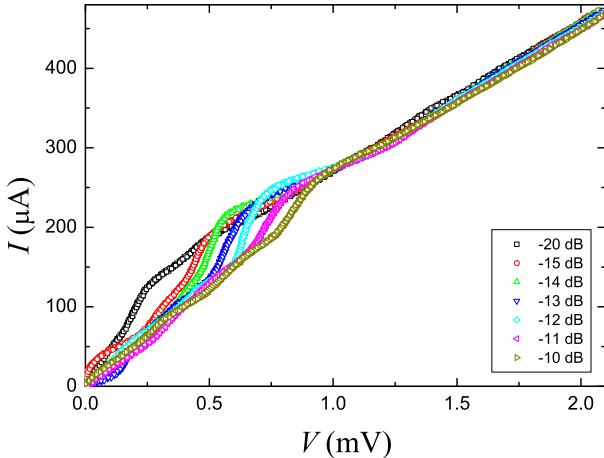


FIG. 3: (Color online) Family of I - V characteristics plotted as a function of incident microwave power level.

To conclude this section, we were able to estimate the rf current amplitude for a wide range of dimensionless parameter ξ , which depends on the junction parameters and the rf frequency. In particular, it was shown that the rf amplitudes required by the theory could be easily obtained in our experimental setup.

V. I - V MEASUREMENTS — RESULTS AND DISCUSSION

In order to study the influence of microwave radiation on the static properties of a Josephson junction, we have performed a series of detailed I - V measurements for various levels of microwave power. Twelve sets of junctions have been measured with the parameters close to those given in Section II.

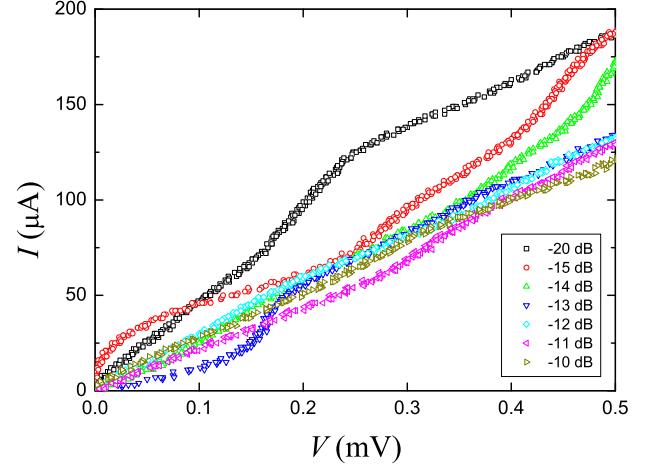


FIG. 4: (Color online) Enlarged part of Fig. 3 plotted for smaller values of current and voltage.

The results obtained so far exhibit a rich variety of different phenomena which are still under investigation. Here we present only a few examples, dealing mainly with the dynamical resistance of the junction in the presence of strong microwave radiation.

A typical example of the I - V characteristic measured for the sample (a) is presented in Fig. 3, while Fig. 4 shows an enlarged part of the same diagram plotted for smaller current and voltage values. According to Section IV, the power level $P/P_0 = -20$ dB corresponds to the absolute rf amplitude $I_a \simeq 150 \mu\text{A}$ and consequently -10 dB denotes $I_a \simeq 480 \mu\text{A}$. For the set of parameters given by Eq. (4), we have $\Omega_1 < 1$ and $\Omega_1 > \sigma$. As a consequence, the Shapiro steps are not observed in this region, in agreement with Refs. 5,6.

It is clear that the I - V characteristics are influenced strongly by the microwave radiation. First, we observe a step in the I - V curve, which corresponds to a local decrease of the dynamical resistance $R_d = dV/dI$ from an asymptotic value $R_d \simeq 5.3 \Omega$ to $R_d \simeq 1.1 \Omega - 1.5 \Omega$. The voltage position of this step, denoted by V_s , depends linearly on the rf amplitude as shown in Fig. 5.

The second effect of the microwave radiation is its influence on the dynamical resistance R_d at zero voltage (see Fig. 6). This time we observe rather large and irregular changes of R_d ranging from a very small value $R_d < 0.1 \Omega$ for $(P/P_0)^{1/2} = 0.177$ up to $R_d \simeq 8.4 \Omega$ for $(P/P_0)^{1/2} = 0.224$. It is clear that the dependence of R_d on the rf radiation is non-monotonic and for $(P/P_0)^{1/2} = 0.177$ we observe nearly zero value of the dynamical resistance. Unfortunately, contrary to theoretical predictions, so far we were not able to observe a negative dynamical resistance of the Josephson junction.

It seems that the main reason for a visible discrepancy between theoretical and experimental results follows from the method used to reduce the critical current of the junction. A more detailed analysis shows¹² that applying an external magnetic field in the junction plane gives rise

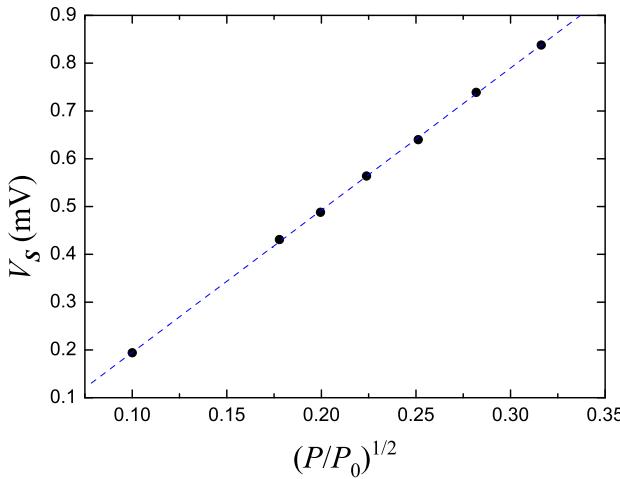


FIG. 5: (Color online) Step position V_s as a function of the relative rf amplitude. Experimental data are plotted as full circles and the dashed line shows a linear fit.

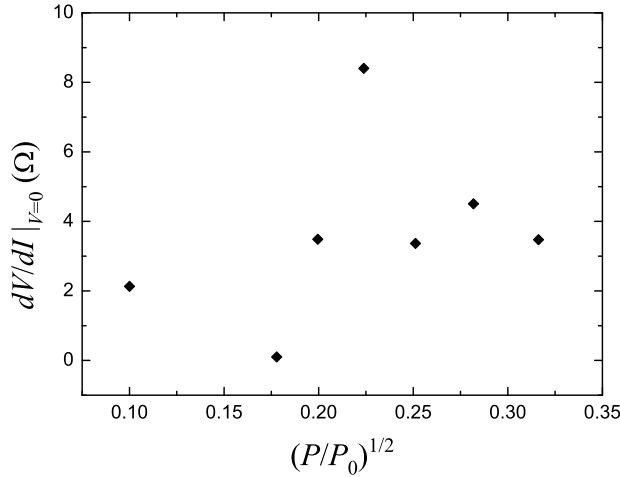


FIG. 6: (Color online) Dynamical resistance at zero voltage as a function of the relative rf amplitude.

to a spatial modulation of the current density. In other words, the external magnetic field makes the phase ϕ to become a function of spatial coordinates, in spite of the fact that we deal with small junctions whose physical dimensions are much smaller than the Josephson penetration depth λ_J . As a consequence, the Stewart-McCumber model becomes inadequate and should be generalized to a multidimensional case, taking the form of a partial differential (sine-Gordon like) equation:

$$-\alpha \nabla^2 \phi + \frac{d^2 \phi}{dt^2} + \sigma \frac{d\phi}{dt} + \sin(\phi) = i_0 + i_1 \cos(\Omega t) + \sqrt{2\sigma D} \Gamma(t) \quad (11)$$

where $\nabla^2 \phi = d^2 \phi / dx^2 + d^2 \phi / dy^2$ denotes the two-dimensional Laplacian, α is a scaling parameter and $\phi(x, y, t)$ is a function of time and two spatial coordinates.

Unfortunately, such a model has no counterpart in the domain of chaotic dynamical systems, hence the theoretical predictions resulting from the Newton-Langevin equation cannot be directly applied to a Josephson junction immersed in an external magnetic field.

VI. SUMMARY AND CONCLUSIONS

In this work we investigated the influence of high-power microwave radiation on the static characteristics of a small Josephson junction. In particular, the dependence of the critical current on the microwave power level was studied, allowing to determine the absolute value of the rf (microwave) current flowing across the junction. It was shown that for the junction parameters as given in Section II, a “current source” model can be applied, leading to a simple relation $I_a \simeq I_0$ at the point where the maximum constant current I_{max} attains zero.

In the next step, a dynamical resistance of the junction was investigated as a function of the rf current amplitude. In particular, it was found that the dynamical resistance at zero voltage is strongly dependent on the microwave power level and may vary in a wide range, however negative values of the resistance were not observed.

As far as the correspondence between Newton-Langevin and Stewart-McCumber models is concerned, we found that the junction parameters required by the theory were incompatible with those of really existing Josephson junctions due to a natural constraint on the product of R_n and I_0 . As pointed out in Section V, an external magnetic field applied in the junction plane may reduce the critical current, but also makes the superconducting current density to be spatially modulated. In other words, the analogy between the NL and SM models breaks down, since the dynamics of a Josephson junction immersed in an external magnetic field should be described by a more general multidimensional equation.

Acknowledgments

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